The fundamental BLUE equation in linear models revisited

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Abstract

In the world of linear statistical models there is a particular matrix equation, $\mathbf{G}(\mathbf{X} : \mathbf{V}\mathbf{X}^{\perp}) = (\mathbf{X} : \mathbf{0})$, which is sufficiently important that it is sometimes called the fundamental BLUE equation. In this equation, $\mathbf{X}$ is a model matrix, $\mathbf{V}$ is the covariance matrix of $\mathbf{y}$ in the linear model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, and we are interested in finding the best linear estimator, BLUE, of $\mathbf{X}\boldsymbol{\beta}$. Any solution $\mathbf{G}$ for this equation has the property that $\mathbf{G}\mathbf{y}$ provides a representation for the BLUE of $\mathbf{X}\boldsymbol{\beta}$: this is the message of the fundamental BLUE equation, whose main developer was the late Professor C. R. Rao in early 1970s.

This talk is part of joint work with Stephen J. Haslett, Jarkko Isotalo and Augustyn Markiewicz.

Keywords

BLUE, BLUP, covariance matrix, equality of the BLUEs, linear sufficiency, misspecified model.

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