Double exponential quadrature in calculating the distribution of products and quotients of random variables

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Abstract

We explore the application of double exponential (DE) quadrature [1, 2] for fast and exact calculations of the distribution of products and quotients of independent random variables, which are important not only in linear statistical inference and uncertainty estimation in metrology but also in other fields. The DE quadrature, which uses the trapezoidal rule with a DE transformation, is employed for the numerical integration of the required probability density expressed as the Mellin convolution integral [3]. In our numerical explorations, we also compare and cross-check our results with well-known analytic formulas [3], recent numerical approaches [4], and Monte Carlo and bootstrap simulations. Since the DE quadrature is well-suited for CPU and GPU parallelization [5], HPC can also be effectively utilized and explored.

Keywords

Mellin convolution integral, exact probability calculations, fast numerical integration, high-performance computing (HPC), numerical simulations

Acknowledgements

This work was supported by the Slovak Research and Development Agency under the Contract No. APVV-21-0369, No. APVV-21-0216 and by the Slovak Scientific Grant Agency VEGA under grant VEGA 1/0585/24.

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